4.2 Irrational Numbers

LESSON FOCUS Identify and order irrational numbers.

Make Connections

The formulas for the area and circumference of a circle involve π, which is not a rational number because it cannot be written as a quotient of integers.

What other numbers are not rational?

Friday, September 23rd

- Review Number Terminology
- Finish notes/examples
- Practice terminology (chart)
- Number Activity
- Classwork

• Definitions

- Real numbers (R): ALL numbers; rational & irrational
- Irrational numbers ($\overline{Q}$):
  - -they cannot be written as a fraction
  - -non-repeating decimal
  - -non-terminating decimal
  - -Examples: 0.2163875943.... and π
- Rational numbers (Q):
  - -a number that can be written as a fraction
  - -Any number that is not an irrational number
  - -Examples: -2.34, $3.4\overline{56}$, 6.323 232 32...

Definitions continued...

- Integers (I):
  - -Positive and negative whole numbers
  - -NO decimals
  - -Examples: -400, +8, 0, 29, -49578
- Whole numbers (W):
  - -all of the positive integers and zero
  - -Examples: 0, 1, 2, 3, 4, etc.
  - -NO decimals
- Natural numbers (N):
  - -all of the positive integers
  - -DOES NOT include zero (only difference from whole numbers)
  - -Examples: 1, 2, 3, 4, etc.

THE NUMBER SYSTEM

W =	Whole Numbers	I =	Integers
$\overline{Q}$ =	Irrational Numbers	R =	Real Numbers
N =	Natural Numbers	Q =	Rational Numbers

EXAMPLES:

W: 0, 1, 2, 3,.....

$\overline{Q}$: π (3.141592…), $\sqrt{3}$, 1.23456738… , $\sqrt{15}$, …

N: 1, 2, 3,.....

I: .....,-3, -2, -1, 0, 1, 2, 3, …

R: $-\frac{1}{2}$, $\sqrt{15}$, 0 , -3 , 3 , π (3.141592) , …

Q: $\frac{1}{2}$, $-\frac{1}{2}$, $\frac{11}{3}$, 0.2 , -0.2 , 3 , -3 , 0 , …

Copy and complete the table:

For each of the following numbers in the table, put an "x" in each category that the number belongs to. It may only belong in one, but could also belong to 5 out of the 6 categories. The first one is done for you.

Number	Real	Rational	Irrational	Whole	Natural	Integer
3.2	x	x				
0						
5.66						
-7						
15						
20009						
4.569…						
3.14…						
-3.22						
4/5						
14/2						
-6/3						
5/2						
-4.567…						
-23						
10						

Copy and complete the table:

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5/2						
-4.567...						
-23						
10						

TRY THIS

Work with a partner.

These are rational numbers.	These are not rational numbers.
$\sqrt{100}$ $\sqrt{0.25}$ $\sqrt[3]{8}$ 0.5	$\sqrt{0.24}$ $\sqrt[3]{9}$ $\sqrt{2}$
$\frac{5}{6}$ $\sqrt{\frac{9}{64}}$ 0.8^2 $\sqrt[5]{-32}$	$\sqrt{\frac{1}{3}}$ $\sqrt[4]{12}$

A. How are radicals that are rational numbers different from radicals that are not rational numbers?

B. Which of these radicals are rational numbers? Which are not rational numbers? How do you know?

$\sqrt{1.44}$, $\sqrt{\frac{64}{81}}$, $\sqrt[3]{-27}$, $\sqrt{\frac{4}{5}}$, $\sqrt{5}$

C. Write 3 other radicals that are rational numbers. Why are they rational?

D. Write 3 other radicals that are not rational numbers. Why are they not rational?

4.2 Irrational Numbers

Radicals that are square roots of perfect squares, cube roots of perfect cubes, and so on are rational numbers. Rational numbers have decimal representations that either terminate or repeat.

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When an irrational number is written as a radical, the radical is the *exact value* of the irrational number; for example, $\sqrt{2}$ and $\sqrt[3]{-50}$. We can use the square root and cube root keys on a calculator to determine *approximate values* of these irrational numbers.

?

?

There are other irrational numbers besides radicals; for example, π.

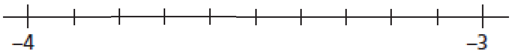
We can approximate the location of an irrational number on a number line. If we do not have a calculator, we use perfect powers to estimate the value. For example, to locate $\sqrt[3]{-50}$ on a number line, we know that $\sqrt[3]{-27} = -3$ and $\sqrt[3]{-64} = -4$.

Guess: $\sqrt[3]{-50} \doteq -3.6$ Test: $(-3.6)^3 = -46.656$

Guess: $\sqrt[3]{-50} \doteq -3.7$ Test: $(-3.7)^3 = -50.653$

This is close enough to represent on a number line.

Since $(-3.7)^3 = -50.653$, then $\sqrt[3]{-50}$ is slightly greater than -3.7, so mark a point to the right of -3.7 on the number line.



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Number line Activity

With a partner, or by yourself, place the numbers from LEAST to GREATEST.

Let's see who can get done first!

(When you are done, call me over so that I can check it)

33	$\sqrt{9}$	1.45	-1.23	-4.5
$\sqrt[3]{13}$	0	$\sqrt{36}$	$\sqrt[3]{-5}$	π
2/3	-8/4	-3	$(5)^3$	8/3
$(-2)^3$	$\sqrt{144}$	1002	-5.68	$\sqrt{64}$
$\sqrt{18}$	9 1/2	-1.22	6/7	$2 \times 4 \div \sqrt{4}$

$(-2)^3$ | -5.68 | -4.5 | -3 | -8/4 | $\sqrt[3]{-5}$ | -1.23 | -1.22 | 0 | 2/3 | 6/7 | 1.45

$\sqrt[3]{13}$ | 8/3 | $\sqrt{9}$ | π | $2 \times 4 \div \sqrt{4}$ | $\sqrt{18}$ | $\sqrt{36}$ | $\sqrt{64}$ | 9 1/2 | $\sqrt{144}$ | 33 | $(5)^3$ | 1002

1. Choose only the rational numbers
2. Choose only the whole numbers
3. Choose only the integers
4. Choose only the irrational numbers
5. Choose only the natural numbers
6. Choose only the real numbers

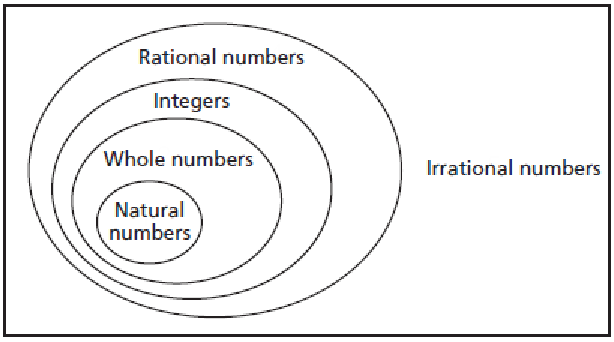
Monday, September 26th

- Review Number Terminology
- Review how to determine if a number is rational or irrational
- Classwork

Review of Number Terminology

Together, the rational numbers and irrational numbers form the set of **real numbers**. This diagram shows how these number systems are related.

Real Numbers



4.2 Irrational Numbers

Task:

Please determine the first 10 perfect squares and perfect cubes:

$\sqrt{1} = 1$ $\sqrt[3]{1} = 1$

How are radicals that are rational numbers different from radicals that are not rational numbers?

4.2 Irrational Numbers

Rational or Irrational???

If you are finding the square root of a number, the number must be a perfect square in order to be a rational number

$\sqrt{16}$ (Rational) $\sqrt{14}$ (Irrational)

If you are finding the cubed root of a number, then the number must be a perfect cube in order to be a rational number.

$\sqrt[3]{27}$ (Rational) $\sqrt[3]{20}$ (Irrational)

$\sqrt[3]{\frac{8}{27}}$ (Rational) $\sqrt{\frac{4}{5}}$ (Irrational)

$\sqrt[3]{-27} = -3$

Which of these radicals are rational numbers?
Which are not rational numbers? How do you know?

$\sqrt{1.44}$ $= 1.2$ $\quad \sqrt{\frac{64}{81}}$ $= \frac{8}{9}$ $\quad \sqrt[3]{-27}$ $= -3$ $\quad \sqrt{\frac{4}{5}}$ $\quad \sqrt{5}$

Write 3 other radicals that are rational numbers. Why are they rational?

Write 3 other radicals that are not rational numbers. Why are they not rational?

Example 1 Classifying Numbers

Tell whether each number is rational or irrational. Explain how you know.

a) $-\frac{3}{5}$ b) $\sqrt{14}$ c) $\sqrt[3]{\frac{8}{27}}$ $= \frac{2}{3}$

SOLUTION

CHECK YOUR UNDERSTANDING

Example 2 Ordering Irrational Numbers on a Number Line

Use a number line to order these numbers from least to greatest.

$\sqrt[3]{13}$, $\sqrt{18}$, $\sqrt{9}$, $\sqrt[4]{27}$, $\sqrt[3]{-5}$

SOLUTION

CHECK YOUR UNDERSTANDING

Tuesday, September 27th

- Check and go over homework
- Learn how to simplify radical expressions.
- Classwork

Reminder: Quiz this Thursday Sept.29
Test Wed. Oct.5

Pg.211

$\boxed{\#\,3, 4, 5, 7, 10,}$ 13, 15, 16

Perfect Squares

$\sqrt{1} = 1$
$\sqrt{4} = 2$
$\sqrt{9} = 3$
$\sqrt{16} = 4$
$\sqrt{25} = 5$
$\sqrt{36} = 6$
$\sqrt{49} = 7$
$\sqrt{64} = 8$
$\sqrt{81} = 9$
$\sqrt{100} = 10$

Perfect Cubes

$\sqrt[3]{1} = 1$
$\sqrt[3]{8} = 2$
$\sqrt[3]{27} = 3$
$\sqrt[3]{64} = 4$
$\sqrt[3]{125} = 5$
$\sqrt[3]{216} = 6$
$\sqrt[3]{343} = 7$
$\sqrt[3]{512} = 8$
$\sqrt[3]{729} = 9$
$\sqrt[3]{1000} = 1$

3. Tell whether each number is rational or irrational.

a) $\sqrt{12}$ $\bar{Q}$ b) $\sqrt[4]{16}$ Q

c) $\sqrt[3]{-100}$ $\bar{Q}$ d) $\sqrt{\frac{4}{9}}$ Q

e) $\sqrt{1.25}$ $\bar{Q}$ f) 1.25 Q

4. Classify each number below as:

a) a natural number b) an integer

c) a rational number d) an irrational number

$\frac{4}{3}$, $0.3\overline{4}$, -5, $\sqrt[4]{9}$, -2.1538, $\sqrt[3]{27}$, 7

a) 7, $\sqrt[3]{27}$

b) 7, $\sqrt[3]{27}$, -5

c)

B

5. a) Why are $\sqrt{49}$ and $\sqrt[4]{16}$ rational numbers? $= 2$

b) Why are $\sqrt{21}$ and $\sqrt[3]{36}$ irrational numbers?

3. Tell whether each number is rational or irrational.

a) $\sqrt{12}$ $\bar{Q}$ b) $\sqrt[4]{16}$ Q

c) $\sqrt[3]{-100}$ $\bar{Q}$ d) $\sqrt{\frac{4}{9}}$ Q

e) $\sqrt{1.25}$ $\bar{Q}$ f) 1.25 Q

4. Classify each number below as:

a) a natural number b) an integer

c) a rational number d) an irrational number

$\frac{4}{3}$, $0.3\overline{4}$, -5, $\sqrt[4]{9}$, -2.1538, $\sqrt[3]{27}$, 7

a) $\sqrt[3]{27}$, 7

b) 7, -5, $\sqrt[3]{27}$

c) $\frac{4}{3}$, $0.3\overline{4}$, -5, $\sqrt[3]{27}$, 7, -2.1538

d) $\sqrt[4]{9}$

B

5. a) Why are $\sqrt{49}$ and $\sqrt[4]{16}$ rational numbers?

b) Why are $\sqrt{21}$ and $\sqrt[3]{36}$ irrational numbers?

49 is a perfect square

$\sqrt[4]{16} = 2$

7. a) Sketch a diagram to represent the set of rational numbers and the set of irrational numbers.

b) Write each number that follows in the correct set.

$\frac{1}{2}$, $-\sqrt{3}$, $\sqrt{4}$, $\sqrt[4]{5}$, $-\frac{7}{6}$, $\sqrt[3]{8}$, 10.12, $-13.\overline{4}$, $\sqrt{0.15}$, $\sqrt{0.16}$, 17

Rat	Irr
$\sqrt{0.16}$, 17, $\frac{1}{2}$, $\sqrt{4}$, $\sqrt[3]{8}$	$-\sqrt{3}$, $4\sqrt{5}$, $\sqrt{0.15}$

7. a) Sketch a diagram to represent the set of rational numbers and the set of irrational numbers.

b) Write each number that follows in the correct set.

$\frac{1}{2}$, $-\sqrt{3}$, $\sqrt{4}$, $\sqrt[4]{5}$, $-\frac{7}{6}$, $\sqrt[3]{8}$, 10.12, $-13.\overline{4}$, $\sqrt{0.15}$, $\sqrt{0.16}$, 17

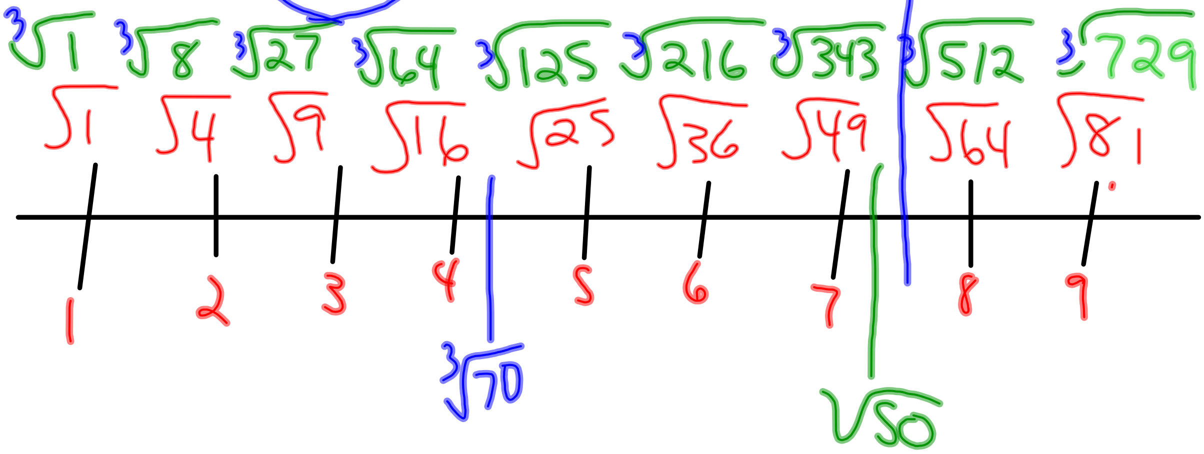
Rational	Irrational
$\frac{1}{2}$, $\sqrt{4}$, $-\frac{7}{6}$, $\sqrt[3]{8}$, 10.12, $-13.\overline{4}$, $\sqrt{0.16}$, 17	$-\sqrt{3}$, $\sqrt[4]{5}$, $\sqrt{0.15}$

10. Use a number line to order the irrational numbers in each set from greatest to least.

a) $\sqrt[3]{\cancel{7}0}, \sqrt{\cancel{5}0}, \sqrt[4]{100}, \sqrt[3]{400}$

b) $\sqrt{89}, \sqrt[4]{250}, \sqrt[3]{-150}, \sqrt[3]{150}$

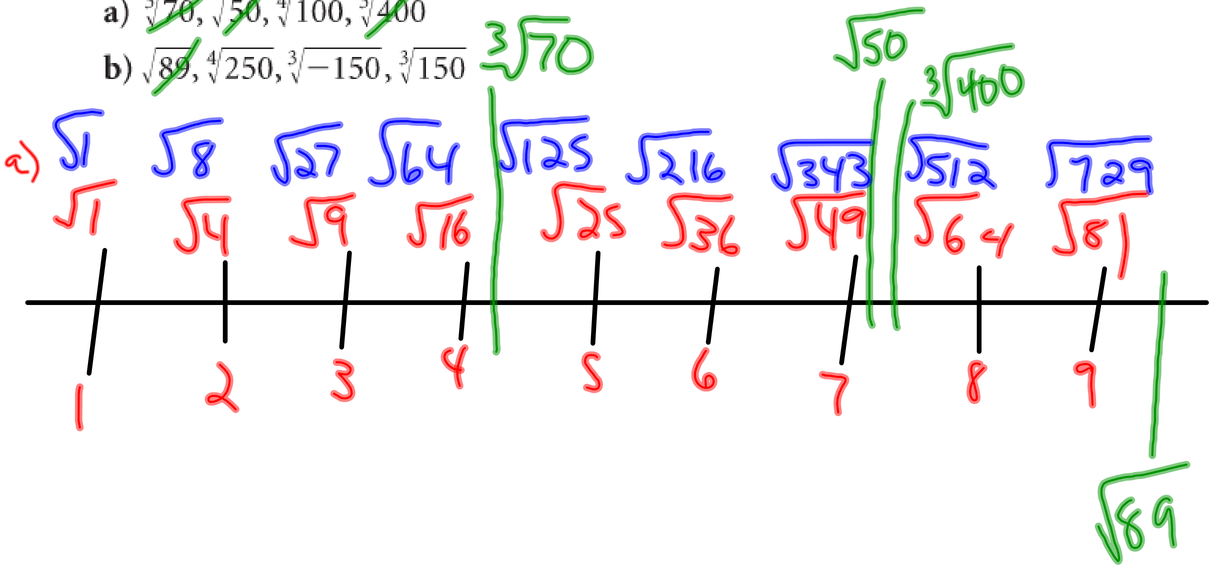
$\sqrt[3]{400}$



10. Use a number line to order the irrational numbers in each set from greatest to least.

a) $\cancel{\sqrt[3]{70}}, \cancel{\sqrt{50}}, \sqrt[4]{100}, \cancel{\sqrt[3]{400}}$

b) $\cancel{\sqrt{89}}, \sqrt[4]{250}, \sqrt[3]{-150}, \sqrt[3]{150}$ $\sqrt[3]{70}$



Discuss the Ideas

1. How do you determine whether a radical represents a rational or an irrational number? Use examples to explain.
2. How can you determine whether the decimal form of a radical represents its exact value?

End of lesson

1 Are these numbers rational or irrational?

Rational

Irrational

$\sqrt{89}$

$\sqrt{2}$

$\dfrac{22}{7}$

Example 1 Classifying Numbers

Tell whether each number is rational or irrational. Explain how you know.

a) $-\frac{3}{5}$ b) $\sqrt{14}$ c) $\sqrt[3]{\frac{8}{27}}$

SOLUTION

a) $-\frac{3}{5}$ is rational since it is written as a quotient of integers.

Its decimal form is -0.6, which terminates.

b) $\sqrt{14}$ is irrational since 14 is not a perfect square.

The decimal form of $\sqrt{14}$ neither repeats nor terminates.

c) $\sqrt[3]{\frac{8}{27}}$ is rational since $\frac{8}{27}$ is a perfect cube.

$\sqrt[3]{\frac{8}{27}} = \frac{2}{3}$ or $0.\overline{6}$, which is a repeating decimal

CHECK YOUR UNDERSTANDING

Example 2 Ordering Irrational Numbers on a Number Line

Use a number line to order these numbers from least to greatest.

$\sqrt[3]{13}$, $\sqrt{18}$, $\sqrt{9}$, $\sqrt[4]{27}$, $\sqrt[3]{-5}$

SOLUTION

13 is between the perfect cubes 8 and 27, and is closer to 8.

$$\begin{array}{ccc} \sqrt[3]{8} & \sqrt[3]{13} & \sqrt[3]{27} \\ \downarrow & \downarrow & \downarrow \\ 2 & ? & 3 \end{array}$$

Use a calculator.

$\sqrt[3]{13} = 2.3513\ldots$

3ˣ√(13)
2.351334688

18 is between the perfect squares 16 and 25, and is closer to 16.

$$\begin{array}{ccc} \sqrt{16} & \sqrt{18} & \sqrt{25} \\ \downarrow & \downarrow & \downarrow \\ 4 & ? & 5 \end{array}$$

(Solution continues.)

Example 2 Ordering Irrational Numbers on a Number Line

Use a calculator.

$\sqrt{18} = 4.2426\ldots$

√(18)
4.242640687

$\sqrt{9} = 3$

27 is between the perfect fourth powers 16 and 81, and is closer to 16.

$$\begin{array}{ccc} \sqrt[4]{16} & \sqrt[4]{27} & \sqrt[4]{81} \\ \downarrow & \downarrow & \downarrow \\ 2 & ? & 3 \end{array}$$

Use a calculator.

$\sqrt[4]{27} = 2.2795\ldots$

4ˣ√(27)
2.279507057

-5 is between the perfect cubes -1 and -8, and is closer to -8.

$$\begin{array}{ccc} \sqrt[3]{-1} & \sqrt[3]{-5} & \sqrt[3]{-8} \\ \downarrow & \downarrow & \downarrow \\ -1 & ? & -2 \end{array}$$

Use a calculator.

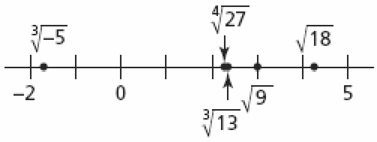
$\sqrt[3]{-5} = -1.7099\ldots$

3ˣ√(-5)
-1.709975947

(Solution continues.)

Example 2 Ordering Irrational Numbers on a Number Line

Mark each number on a number line.



From least to greatest: $\sqrt[3]{-5}, \sqrt[4]{27}, \sqrt[3]{13}, \sqrt{9}, \sqrt{18}$

CHECK YOUR UNDERSTANDING

CHECK YOUR UNDERSTANDING

1. Tell whether each number is rational or irrational. Explain how you know.

   a) $\sqrt{\frac{49}{16}}$ b) $\sqrt[3]{-30}$ c) 1.21

CHECK YOUR UNDERSTANDING

2. Use a number line to order these numbers from least to greatest.
$\sqrt{2}$, $\sqrt[3]{-2}$, $\sqrt[3]{6}$, $\sqrt{11}$, $\sqrt[4]{30}$

Discuss the Ideas

1. How do you determine whether a radical represents a rational or an irrational number? Use examples to explain.

Discuss the Ideas

2. How can you determine whether the decimal form of a radical represents its exact value?

How can you order a set of irrational numbers if you do not have a calculator?